THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMATH5220 Complex Analysis and Its Applications 2014-2015
Assignment 3

- Due date: 18 Mar , 2015
- Remember to write down your name and student number

1. If C is the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate each of these integrals:
   (a) $\displaystyle\int_C \frac{e^{-z}}{z - (\pi i/2)}\, dz$
   (b) $\displaystyle\int_C \frac{\cos z}{z(z^2+8)}\, dz$

2. Let $n \in \mathbb{Z}$ and C be the positively oriented unit circle. Compute
$$\int_C \frac{e^z}{z^n}.$$
(Hints: There are two cases.)

3. Let f be an entire function.
   (a) If $f^{(n)}(z) \equiv 0$ for some natural number n, show that $f(z)$ is a polynomial.
   (b) Prove that if $|f(z)| < |z|^n$ for all $|z| > R$, where $R > 0$ and n is a natural number, then $f(z)$ must be a polynomial. (Hint: Using Cauchy integral formula to estimate $f^{(n+1)}(z)$.)

4. By integrating the function
$$\frac{1}{z}\left(z + \frac{1}{z}\right)^{2n}$$
around the unit circle, parametrized by the curve $\gamma(t) = e^{it}$, $0 \le t \le 2\pi$, show that for any natural number n,
$$\frac{1}{2\pi}\int_0^{2\pi} \cos^{2n} t dt = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)}.$$

5. Expand e^z into a Taylor series about the point $z = 1$.

6. With the aid of series, prove that the function f defined by
$$f(z) = \begin{cases} \dfrac{e^z - 1}{z} & \text{if} \quad z \neq 0 \\ 1 & \text{if} \quad z = 0 \end{cases}$$
is an entrie function.